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Approach for incorporating narrow band nonuniformity into nongray analysis of radiative heat transfer in nonisothermal and nonhomogeneous gas fields

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Abstract

This paper presents a rational, explicit and practicable approach for incorporating the spectral nonuniformity within a narrow band into a nongray analysis of radiative heat transfer. First of all, emphasis is placed on describing the necessity of a new approach for treating this kind of spectral nonuniformity. In the development of this new approach, attention is paid to securing rationality and explicitness even when the approach is applied to nonisothermal fields. This aim is achieved based on the fact that the line center of each absorption line does not move even if the gas temperature varies. Validity of the proposed approach is demonstrated in a typical nonisothermal field. Furthermore, the extension of the proposed approach to the mixture of more than one infrared-active species is discussed and its validity is examined.

Keywords: Radiative heat transfer; Infrared-active gas; Narrow band nonuniformity; Nongray analysis; Nonisothermal and nonhomogeneous fields

1. Introduction

Absorption coefficient of an infrared-active gas varies so strongly and rapidly across the spectrum that the assumption of a gray gas is almost never a good one [1]. In other words, a nongray analysis is required for properly evaluating the radiative heat transfer through an infrared-active gas. The so-called line-by-line analysis would be the most rigorous way for a nongray analysis. Even the computers of nowadays, however, are not sufficient for carrying out a line-by-line analysis in relation to practical equipment like an industrial furnace, because all the relevant spectral range has to be divided into an enormous number of sub-line bands.

It seems practicable to use some model for dealing with the nongray characteristic of an actual infrared-active gas. Various models have been proposed for this purpose. These models were described already in early books [1–3] as well as recent ones [4,5]. A weighted-sum-of-gray-gas (WSGG) model [2] has been proposed for easily evaluating the emittance at various temperatures, pressures and optical depths. Also the trial for constructing a WSGG model based on the HITRAN [6] database has been reported [7,8]. However, these WSGG models are valid only when the absorption coefficient spectrum in the relevant spectral range keeps a similar pattern all over the relevant spatial domain. Such a condition is, however, hardly fulfilled if the field is nonisothermal or nonhomogeneous.

It would be eventually conceded that a nongray analysis based on some band model is a rational and realistic way from the practical point of view when the field is considerably nonisothermal and nonhomogeneous like in a furnace. In this kind of approach,

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the relevant spectral range is divided into a reasonable number of narrow bands. Then, the representative value of absorption coefficient in each narrow band is evaluated based on the wide band profile assumed in an exponential wide band model, for example. The nongray characteristic of an infrared-active gas is partly incorporated through this approach. However, each narrow band still contains many absorption lines, and the absorption coefficient spectrum shows oscillatory behavior even in a narrow band. Of course, also this kind of spectral variation of absorption coefficient has to be taken into consideration for carrying out a proper nongray analysis when there is considerable unevenness within a narrow band. In this paper, this kind of unevenness of absorption coefficient within a narrow band is referred to as "narrow band nonuniformity".

The Lambert–Beer law no longer holds well when the absorption coefficient spectrally averaged over many lines is used. But, it seems to be expected that narrow band models help us to carry out a nongray analysis taking the narrow band nonuniformity into consideration without further spectral division of each narrow band. Such expectation may appear, because there are some algebraic formulae derived from the narrow band models and the emittance, transmittance or absorptance spectrally averaged over a narrow band is easily evaluated as a function of the averaged absorption coefficient, line overlapping parameter, path length and so on.

In spite of this expectation, we should pay attention to a hard fact that the transmittance spectrally averaged over a narrow band depends on the incident power spectrum in the relevant narrow band. Moreover, we need the value of transmittance and not of emittance for simulating radiative heat transfer. Namely, for carrying out a nongray analysis taking the narrow band nonuniformity into consideration, we need an approach where the dependency of the transmittance averaged over a narrow band on the incident power spectrum is properly simulated. However, except for an impracticable approach of a line-by-line analysis, there has not been a rational and explicit approach that satisfies this request.

In the first half of this paper, emphasis is placed on the necessity of a new approach for incorporating narrow band nonuniformity into the simulation of radiative heat transfer. So, it is illustrated that disregarding the narrow band nonuniformity often leads to a serious error in a simulation of radiative heat transfer, and moreover, the transmittance spectrally averaged over a narrow band severely depends on the incident power spectrum in the relevant narrow band.

In the latter half, this paper proposes a rational, explicit and practicable approach for incorporating the narrow band nonuniformity into the simulation of radiative heat transfer. Firstly, fundamental concept is explained in relation to the isothermal fields. Secondly, the approach is extended to the nonisothermal fields and its validity is examined by comparing its results with the results based on the Curtis–Godson approximation [9]. Also the expansion of the proposed approach to a gas mixture including more than one infrared-active species is described and examined.

2. Influence of narrow band nonuniformity

Narrow band nonuniformity becomes remarkable when the total pressure of a gas mixture decreases, and in consequence, the pressure broadening of each absorption line diminishes. If the absorption coefficient profile within a narrow band is treated as quite flat in spite of actual nonuniformity of a considerable level, such simplified treatment may lead to a serious error in a simulation of radiative heat transfer. In this section, we take up water vapor and carbon dioxide, which are the principal infrared-active species in combustion fields, and examine the error induced by disregarding the narrow band nonuniformity on a typical condition of atmospheric pressure.

2.1. Typical situation for examination

For examining the influence of narrow band nonuniformity on radiative heat transfer, total emissive power is considered at the bottom center of a hemispherical mass of an isothermal gas mixture with homogeneous composition. The monochromatic emissive power E_η at this point is given by

$$E_\eta = E_{B\eta}\left(1 - e^{-K_\eta L}\right) = E_{B\eta}\left(1 - e^{-\kappa_\eta pL}\right), \tag{1}$$

where η is the wave number, $E_{B\eta}$ the monochromatic blackbody emissive power (Planck function), L the radius of a hemispherical mass of a gas mixture, p the partial pressure of the infrared-active species, K_η the monochromatic absorption coefficient (length^{-1}), κ_η is the monochromatic pressure absorption coefficient ($= K_\eta/p$).

Because the Planck function is almost constant within a narrow spectral range regarded as a narrow band, the definition of the narrow-band average of spectral emittance can be simplified with good accuracy

$$\varepsilon_{\eta,\mathrm{nb}} = \frac{\int_{\eta-\eta_{\mathrm{nb}}/2}^{\eta+\eta_{\mathrm{nb}}/2} E_{B\eta}\left(1 - e^{-\kappa_\eta pL}\right) d\eta}{\int_{\eta-\eta_{\mathrm{nb}}/2}^{\eta+\eta_{\mathrm{nb}}/2} E_{B\eta}\, d\eta} = \frac{1}{\eta_{\mathrm{nb}}} \int_{\eta-\eta_{\mathrm{nb}}/2}^{\eta+\eta_{\mathrm{nb}}/2} \left(1 - e^{-\kappa_\eta pL}\right) d\eta. \tag{2}$$

In this expression, the symbol η_{nb} denotes the spectral width of a narrow band in terms of wave number, and the subscript nb in the symbol $\varepsilon_{\eta,\mathrm{nb}}$ means that this is the emittance spectrally averaged within a narrow band.

Hereafter, this notation is similarly used also as for the transmittance or absorptance spectrally averaged within a narrow band. And, such spectrally averaged value is written with a modifier "narrow-band" like the "narrow-band emittance".

Integrating Eq. (1) and taking account of the abovementioned characteristic of the Planck function again, we can formulate the total emissive power as follows.

$$E = \int_0^\infty E_\eta \, d\eta = \int_0^\infty E_{B\eta}\left(1 - e^{-\kappa_\eta pL}\right) d\eta = \int_0^\infty E_{B\eta}\left\{\frac{1}{\eta_{nb}}\int_{\eta-\eta_{nb}/2}^{\eta+\eta_{nb}/2}\left(1 - e^{-\kappa_{\eta'} pL}\right) d\eta'\right\} d\eta. \quad (3)$$

From Eqs. (2) and (3), we obtain

$$E = \int_0^\infty E_{B\eta}\varepsilon_{\eta,nb}\, d\eta. \quad (4)$$

Now, the discussion is developed based on the Elsasser model [10]. The distribution of absorption coefficient within a narrow band becomes flat when the ratio of line half-width to line spacing, γ/d (line overlap parameter), increases. Taking the limit $\gamma/d \to \infty$, the distribution becomes quite flat and its value equals to the ratio of line intensity to line spacing, S/d. In such situation, the narrow-band emittance is given by simplifying Eq. (2)

$$\varepsilon_{\eta,nb} = 1 - e^{-(S/d)_\eta pL}, \quad (5)$$

where S is the line intensity (definite integral of the pressure absorption coefficient), d is the line spacing.

From Eqs. (4) and (5), we obtain the formula describing the total emissive power

$$E = \int_0^\infty E_{B\eta}\left[1 - \exp\left\{-(S/d)_\eta pL\right\}\right] d\eta. \quad (6)$$

On the other hand, when the value of γ/d is small, the distribution of absorption coefficient in a narrow band displays violent oscillation and Eq. (6) no longer holds good. Applying the Elsasser model to incorporate the narrow band nonuniformity, Eq. (2) is transformed to

$$\varepsilon_{\eta,nb} = 1 - \frac{1}{2\pi}\int_{-\pi}^{\pi} \exp\left\{-\left(\frac{S}{d}\right)_\eta \cdot \frac{X\sinh\beta_\eta}{\cosh\beta_\eta - \cos z}\right\} dz. \quad (7)$$

From Eqs. (4) and (7), we obtain

$$E = \int_0^\infty E_{B\eta}\left[1 - \frac{1}{2\pi}\int_{-\pi}^{\pi} \exp\left\{-\left(\frac{S}{d}\right)_\eta \frac{X\sinh\beta_\eta}{\cosh\beta_\eta - \cos z}\right\} dz\right] d\eta, \quad (8)$$

where $\beta_\eta = 2\pi(\gamma/d)_\eta$ (pressure broadening parameter), $X = pL$ (pressure path length), $z = 2\pi\eta/d$, γ = line half-width at half height.

Comparing the total emissive power obtained from Eq. (6) with that from Eq. (8) on a condition that an identical wide band model [11] is used commonly for evaluating the values of $(S/d)_\eta$ and β_η, the influence of narrow band nonuniformity should be clarified.

2.2. Results of evaluation

Whether Eq. (6) gives a good approximation for the total emissive power or not depends on the extent of narrow band nonuniformity of actual infrared-active gas, and the extent of narrow band nonuniformity is closely related to the line overlap parameter. Taking account of the combustion fields, we take up CO_2 and H_2O and evaluate the line overlap parameter of their absorption bands based on the relevant formula given as a part of an exponential wide band model [11].

Fig. 1 shows the temperature dependence of line overlap parameter γ/d evaluated on condition that the coexisting gas is nitrogen and the total pressure $p_T = 1$ atm (0.1013 MPa). Taking account of combustion field,

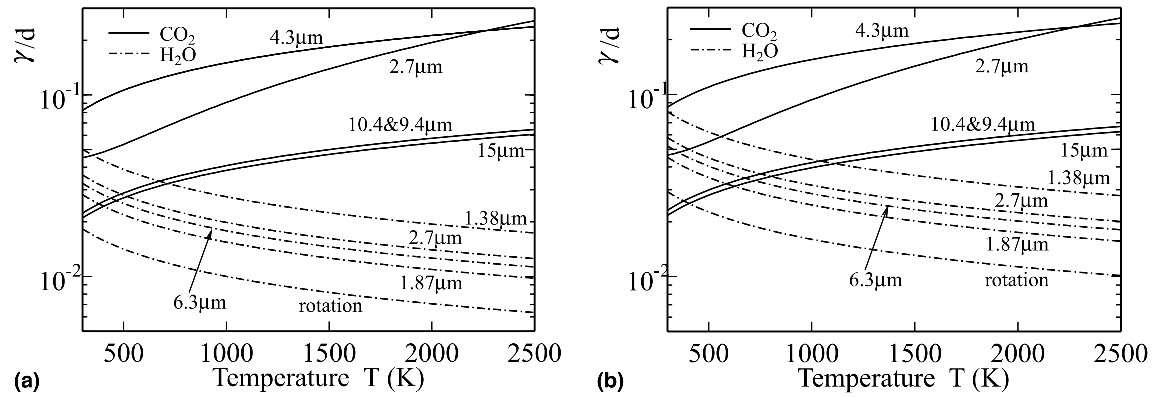


Fig. 1. Line overlap parameter γ/d evaluated for H_2O and CO_2 based on an exponential wide band model [11]: (a) $P_{H_2O/CO_2} \to 0$ atm, $p_T = 1$ atm, (b) $P_{H_2O/CO_2} \to 0.15$ atm, $p_T = 1$ atm.

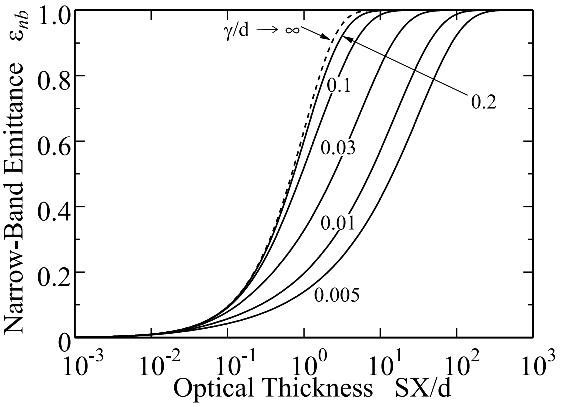


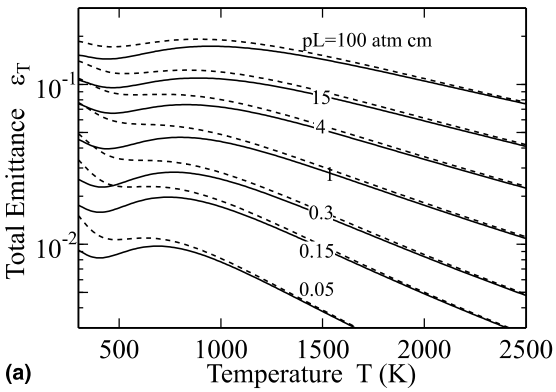
Fig. 2. Effect of line overlap parameter γ/d on the relation between narrow-band emittance and optical thickness (based on the Elsasser narrow-band model).

two kinds of partial pressures ($p_{CO_2/H_2O} \rightarrow 0$ and $p_{CO_2/H_2O} = 0.15$ atm) are taken up. It is noticed from Fig. 1 that the temperature dependence of line overlap parameter for CO_2 is contrary to that for H_2O.

Fig. 2 shows how the relation between the narrow-band emittance and the optical thickness defined as *SX*/*d* depends on the value of line overlap parameter γ/d. These results were obtained from Eq. (7). The curve shown with a broken line is obtained when $\gamma/d \rightarrow \infty$ and corresponds to the disregard of narrow band nonuniformity. The difference between the curve for $\gamma/d = 0.2$ and the broken line curve is small. Furthermore, if γ/d is greater than 0.5, the corresponding curve overlaps the broken line. In other words, if the line overlap parameter is greater than 0.2, disregarding the narrow band nonuniformity does not lead to a serious error in a radiative transfer simulation. Fig. 1 shows, however, that only the 2.7 and 4.3 μm bands of CO_2 fulfill such a condition in a limited high temperature range. The error brought by the disregard of narrow band nonuniformity increases with the decrease in γ/d and considerable error appears even at $\gamma/d = 0.1$. All the absorption bands of H_2O fall below this criterion all over the temperature range (300–2500 K), and also the 15, 10.4 and 9.4 μm bands of CO_2. This fact suggests that the narrow band nonuniformity effect on radiative heat transfer cannot be neglected when the furnace is operated at the atmospheric pressure.

Fig. 3 shows the total emittance ε_T of a gas mixture containing CO_2 or H_2O evaluated at various temperatures and at various pressure path lengths, on condition that the total pressure $p_T = 1$ atm and the partial pressure of a infrared-active species p_{CO_2} or $p_{H_2O} \rightarrow 0$. The total emittance exhibited with solid lines and broken lines is obtained from Eq. (8) or (6), respectively. The values of $(S/d)_\eta$ and β_η appearing in Eqs. (6) and (8) are evaluated based upon an exponential wide band model [11]. The dash–dot lines in Fig. 3(b) correspond to the emittance obtained from the sum of the product of the total band absorptance given by algebraic formulae [11] and the monochromatic blackbody emissive power at representative wave number of each absorption band. It is noticed that the emittance based on total band absorptance is almost the same as that given by Eq. (8). In Fig. 3(a), dash lines are abbreviated because they almost overlap the solid lines. The values of emittance exhibited by the solid lines are near to those found in the Hottel's charts except for those in a low temperature range of H_2O. Good agreement of solid and dash lines suggests that the deviation from the Hottel's chart may be attributed to the modeling of the pure rotation band of H_2O.

In case of H_2O, considerable differences are recognized between the corresponding broken and solid lines all over the range of gas temperature. This indicates that



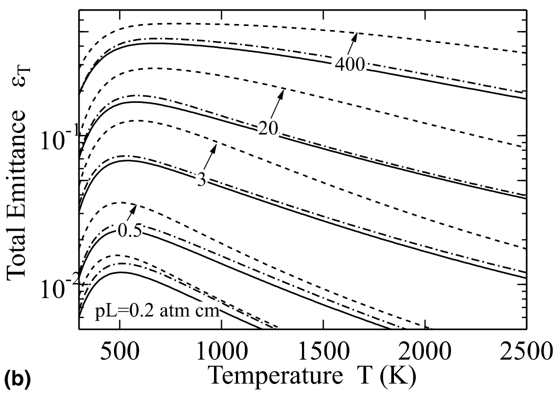


Fig. 3. Effect of narrow-band nonuniformity on the total emittance of CO_2 and H_2O: (a) CO_2 ($p_{CO_2} \rightarrow 0$ atm, $p_T = 1$ atm), (b) H_2O ($p_{H_2O} \rightarrow 0$ atm, $p_T = 1$ atm).

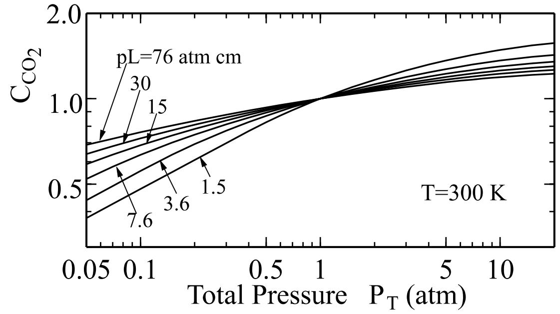


Fig. 4. Correction factor corresponding to the effect of total pressure on the total emittance of CO_2.

the negligence of narrow band nonuniformity leads to a serious error in radiative transfer simulations. Similar remarkable differences are recognized also when the temperature of CO_2 is low.

Fig. 4 shows the total-pressure dependence of total emittance in relation to the gas mixture at $T = 300$ K containing CO_2 and N_2. Total emissive power at arbitrary total pressure is estimated based on Eq. (8) and an auxiliary equation giving the pressure dependence of line overlap parameter γ/d [11], and then, it was normalized by the total emissive power at $p_T = 1$ atm. Fig. 4 shows these normalized values with the symbol C_{CO_2}. When the total pressure is high, line overlap parameter γ/d becomes large. This leads to the increase in the emittance averaged over a narrow band as shown in Fig. 2, and finally leads to the increase in total emittance. When the total pressure is very high, the absorption coefficient spectrum within a narrow band becomes flattened because of the pressure broadening of each absorption line. As a result, the influence of narrow band nonuniformity becomes negligible and the total emittance saturates when the total pressure is very high as shown in Fig. 4. This figure indicates that the disregard of narrow band nonuniformity sometimes gives rise to a considerable error even if the total pressure equals to atmospheric pressure. A more serious error arises when the total pressure is lower than atmospheric pressure.

Since the total emissive power is a summation of the radiative energy transmitted from each part of the relevant mass of gas, erroneous evaluation of emissive power can be attributed to the improper evaluation of narrow-band transmittance induced by disregarding the narrow band nonuniformity. Of course, erroneous evaluation of narrow-band transmittance leads to the error in radiative heat transfer simulation. The results obtained so far suggest that there are many problems where the narrow band nonuniformity must be considered for a proper nongray analysis of radiative heat transfer.

3. Approach for a nongray analysis taking account of narrow band nonuniformity

3.1. Narrow-band transmittance

In the next step, we have to examine whether narrow band models can help us to carry out nongray analysis incorporating narrow band nonuniformity without further spectral division of each narrow band. If we adopt such an approach, we need the value of narrow-band transmittance or narrow-band absorptance, because we have to know the amount of radiative energy absorbed by a certain mass of gas to carry out a simulation of radiative heat transfer. However, narrow-band transmittance depends on the incident power spectrum within the relevant narrow band. Narrow-band emittance is not suitable for this kind of use though it does not depend on the incident power spectrum.

Here, we illustrate how remarkably the narrow-band transmittance depends on the incident spectrum based on the Elsasser narrow band model. When the incident energy spectrum is quite flat over a narrow band, the transmittance spectrally averaged over a narrow band is determined by

$$\tau_{\eta,\mathrm{nb}} = \frac{1}{2\pi}\int_{-\pi}^{\pi} \exp\left\{-\left(\frac{S}{d}\right)_\eta \frac{X\sinh\beta_\eta}{\cosh\beta_\eta - \cos z}\right\}\mathrm{d}z \tag{9}$$

and, shown by the broken lines in Fig. 5 Such flat incident energy spectrum within a narrow band is expected when we pay attention to the radiative energy emitted from a solid wall.

On the contrary, the power spectrum of radiative energy emitted by an infrared-active gas is not uniform within a narrow band. Its spectrum pattern is directly proportional to that of the absorption coefficient. If this same gas absorbs this energy, narrow-band transmittance is given by

$$\tau_{\eta,\mathrm{nb}} = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\sinh\beta_\eta}{\cosh\beta_\eta - \cos z} \times \exp\left\{-\left(\frac{S}{d}\right)_\eta \cdot \frac{X\sinh\beta_\eta}{\cosh\beta_\eta - \cos z}\right\}\mathrm{d}z \tag{10}$$

and shown by the solid lines in Fig. 5.

Comparing the transmittance given by Eq. (10) with that given by Eq. (9), the followings are noticed. The narrow-band transmittance of the radiative energy emitted by infrared-active gas itself decreases more rapidly with the optical path length when the line overlap parameter is small. On the contrary, the narrow-band transmittance of the radiative energy emitted from solid wall decreases more slowly when the line overlap parameter is small. As a result, transmittance of gas emission is always lower than that of wall emission, and the difference between these two cases

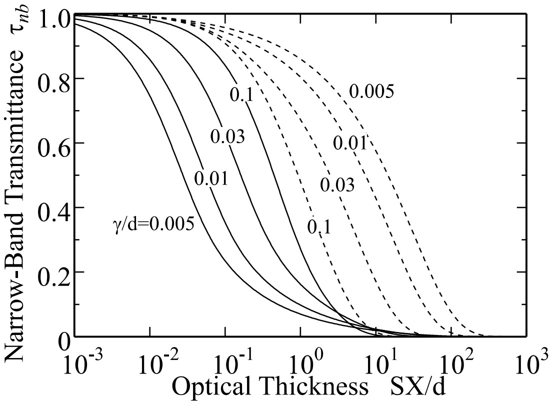


Fig. 5. Effect of incident energy spectrum on the relation between narrow-band transmittance and optical thickness.

is more remarkable when the line overlap parameter is small, that is, when the narrow band nonuniformity is remarkable.

As the narrow-band transmittance strongly depends on the incident power spectrum in the relevant narrow band, we should know the energy spectrum pattern within each narrow band to carry out a simulation of radiative heat transfer in participating gas. Moreover, we have to evaluate the narrow-band transmittance for all the possible spectral pattern of incident energy observed at the interface of calculation grid cells. However, it is quite impossible to make up a perfect algebraic formula that can evaluate the narrow-band transmittance for all the possible spectral pattern of incident energy. And, even such a formula cannot determine the energy spectrum at the end of a certain optical path.

According to the above discussion, we cannot avoid the further spectral division of each narrow band to carry out nongray analysis incorporating the narrow band nonuniformity. However that may be, we cannot carry out a line-by-line analysis over a whole spectral domain in relation to an actual equipment. Therefore, we have to develop a new approach for incorporating the narrow band nonuniformity into radiative heat transfer analysis. In the following part, a rational and explicit approach is proposed for overcoming above-mentioned difficulty related to the narrow band nonuniformity.

3.2. Treatment of narrow band nonuniformity for isothermal case

In the first place, an isothermal and homogeneous field is considered for simplicity. A relevant wave number range has to be divided into a suitable number of narrow bands for taking account of the gentle spectral variation of pressure absorption coefficient, S/d, described by the wide band model. Moreover, as mentioned above, we have to divide each narrow band into a certain number of spectral ranges finer than a narrow band to incorporate the narrow band nonuniformity. On the other hand, we want to suppress the number of spectral division for curtailing the calculation time.

Supposing that the Elsasser model can be applied to each narrow band, the absorption coefficient spectrum within a narrow band is periodical and symmetric about the line center. Hence, the radiative heat transfer within a narrow band can be represented by a half period. Dividing the half period into N sub-narrow-band domains and numbering them from 1 to N in the order of nearness to the line center as shown in Fig. 6, we can determine the absorption coefficient of the sub-narrow-band domain numbered with i as follows (Fig. 7).

$$\kappa_{i,\eta} = \left(\frac{S}{d}\right)_\eta \frac{\sinh \beta_\eta}{\cosh \beta_\eta - \cos\{(\pi/N)(i - 1/2)\}}. \tag{11}$$

Here, the values of S/d and β are evaluated based on an exponential wide-band model [11] at the wave number corresponding to the center of each narrow band. The sub-narrow-band domains having the same absorption coefficient stand at intervals within a narrow band, and their total width in wave number domain amounts to η_{nb}/N (η_{nb}: spectral width of the relevant narrow band). Now, we treat the collection of sub-narrow-band domains having the same absorption coefficient $\kappa_{i,\eta}$ as the No. i group in this narrow band. Arbitrary calculation methods developed for a gray analysis can be applied to each group like this. The calculation load required for this method is fairly light compared with so-called "line-by-line analysis".

Incidentally, when this method is used, the emission rate of radiation energy emitted from No. i group is given by the following formula.

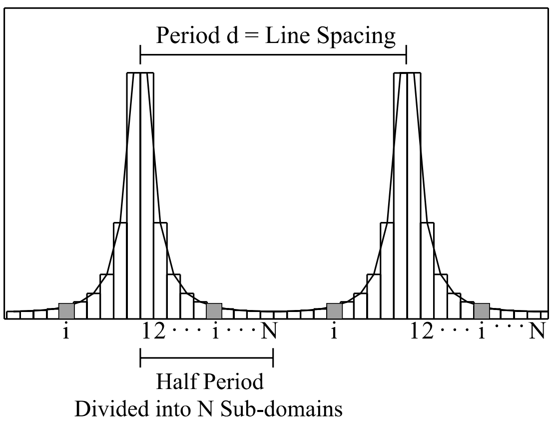


Fig. 6. Concept for spectral division of a narrow band adopted for dealing with the narrow-band nonuniformity in a homogeneous and isothermal field.

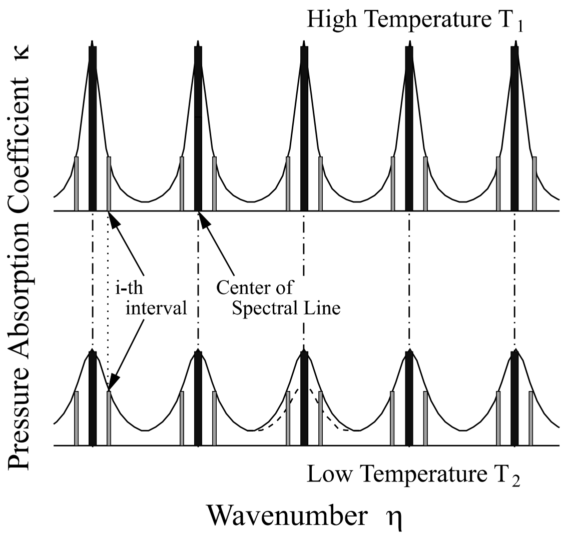


Fig. 7. Concept for determing pressure absorption coefficient used in the proposed approach for simulating the radiative heat transfer in a nonisothermal field.

$$W_V = 4E_{B\eta} \cdot \frac{\eta_{nb}}{N} \cdot \kappa_{i,\eta} p \cdot V, \tag{12}$$

where V is the volume of gas, p is the partial pressure of participating species.

The absorption process of this energy can be traced with the Lambert–Beer law based on the local value of absorption coefficient given by Eq. (11).

3.3. Treatment of narrow band nonuniformity in nonisothermal case

Let us consider the situation where the radiation energy emitted from a gas element at temperature T_1 is absorbed by the same kind of gas at temperature T_2. Paying attention to the No. i group in a certain narrow band, the absorption coefficient of the emitting gas is given by the following formula.

$$\kappa_{i,\eta,T_1} = \left(\frac{S}{d}\right)_{\eta,T_1} \frac{\sinh \beta_{\eta,T_1}}{\cosh \beta_{\eta,T_1} - \cos\{(\pi/N)(i-1/2)\}}. \tag{13}$$

Subscript T_1 means that the values of S/d and β are evaluated at this temperature. To trace the absorption process of this radiation energy, we need the absorption coefficient of this gas at temperature T_2 and at the identical wave number where the radiation energy under consideration is emitted. Since it can be assumed that the line center does not move even if the gas temperature varies, absorption coefficient spectra at different temperature have quite the same period and the locations of peaks in both spectra coincide with each other. This indicates that the absorption coefficient of each sub-narrow-band domain classified to No. i group at temperature T_1 is same and these sub-narrow-band domains can be treated as one group also at temperature T_2. The absorption coefficient needed for calculation is

$$\kappa_{i,\eta,T_2} = \left(\frac{S}{d}\right)_{\eta,T_2} \frac{\sinh \beta_{\eta,T_2}}{\cosh \beta_{\eta,T_2} - \cos\{(\pi/N)(i-1/2)\}}. \tag{14}$$

So far, various solution techniques, such as spherical harmonics technique [12], discrete ordinate technique [13], moment technique [14] and radiative heat ray technique [15], have been proposed for the effective solution of the radiative transfer equation. Though these solution techniques for the radiative transfer equation have been proposed at first for a gray analysis of radiative heat transfer, they can be applied to each group in a narrow band. That is, these solution techniques can be easily combined with a new approach for incorporating the narrow band nonuniformity into a nongray analysis of radiative heat transfer.

3.4. Examination of proposed approach in a nonisothermal field

For examining the validity of the proposed approach, let us consider a stratified nonisothermal gas layer having one-dimensional linear temperature distribution whose maximum and minimum temperature are 2500 and 300 K, respectively. In the numerical examination, this stratified gas is divided into 88 layers of uniform thickness, and the linear temperature distribution in the gas is approximated by a stepwise distribution. It is assumed that a beam of radiative energy, which has a quite flat incident energy spectrum within a narrow band, penetrates this stratified nonisothermal gas. Paying attention to No. i group, the amount of incident energy allocated to this group is $1/N$ of the total incident energy within this narrow band. The amount of energy at the exit of the first layer, whose temperature is T_1, is determined from the absorption coefficient κ_{i,η,T_1} given by Eq. (13) and the pressure path length pL_1 within this layer. Ratio of the energy at the exit of the first layer to that at its inlet is equal to $\exp(-\kappa_{i,\eta,T_1} pL_1)$. Considering the absorption process in each layer similarly and paying attention to the exit of the last layer, the transmittance of the radiation energy belonging to No. i group is given by

$$\tau_{i,\eta} = \exp\left(-p \sum_j \kappa_{i,\eta,T_j} L_j\right). \tag{15}$$

Determining the transmittance for all other groups in this narrow band similarly, and summing them after multiplying with $1/N$, the absorptance averaged over this narrow band is given by

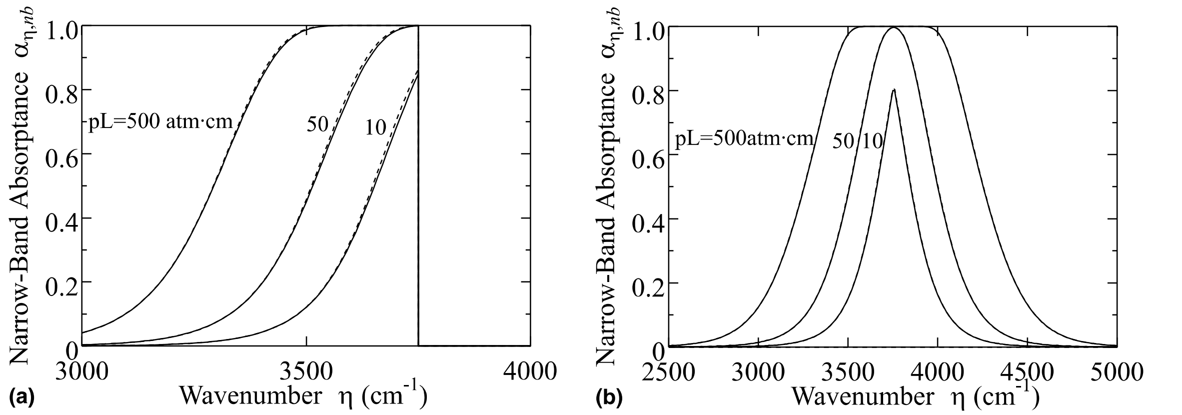


Fig. 8. Comparison of the spectral pattern of narrow-band absorptance in atomospheric nonisothermal field (solid lines: base on the approach proposed in this paper, broken lines: based on the Curtis–Godson approximation): (a) CO_2:2.7 μm band ($p_{CO_2} = p_T = 1$ atm), (b) H_2O:2.7 μm band ($p_{H_2O} = p_T = 1$ atm).

$$\alpha_{\eta,\mathrm{nb}} = 1 - \frac{1}{N}\sum_{i=1}^{N} \exp\left(-p\sum_{j} \kappa_{i,\eta,T_j} L_j \right). \tag{16}$$

Pure CO_2 and pure H_2O at the atmospheric pressure are taken up here as typical examples. The narrow-band absorptance calculated in this way is displayed in Fig. 8 with the solid lines. On the contrary, the absorptance displayed with the broken lines is obtained from the Curtis–Godson approximation, which is considered to be the best approximation so far. Its value is given by

$$\alpha_{\eta,\mathrm{nb}} = 1 - \frac{1}{2\pi}\int_{-\pi}^{\pi} \exp\left\{ -\left(\frac{S}{d}\right)_{\eta,\mathrm{eq}} \cdot \frac{(pL)_{\mathrm{eq}} \sinh\beta_{\eta,\mathrm{eq}}}{\cosh\beta_{\eta,\mathrm{eq}} - \cos z} \right\} \mathrm{d}z, \tag{17}$$

where $(pL)_{\mathrm{eq}}$, $(S/d)_{\mathrm{eq}}$ and β_{eq} are the equivalent values of pressure path length and so on, which are estimated after replacing the stratified gas layer by an equivalent homogeneous gas based on the Curtis–Godson approximation. The difference between the result based on Eqs. (16) and (17) is very scarce, and especially in case of H_2O, it is difficult to distinguish the solid lines from the broken lines.

In case of CO_2, the difference between our approach and the Curtis–Godson approximation becomes more noticeable when the pressure path is short and temperature gradient is steep. Even in the worst case, however, difference is negligible. Fig. 9 shows the similar comparison for the cases where the total pressure is higher or lower than atmospheric pressure. These results indicate that the validity of the proposed approach for

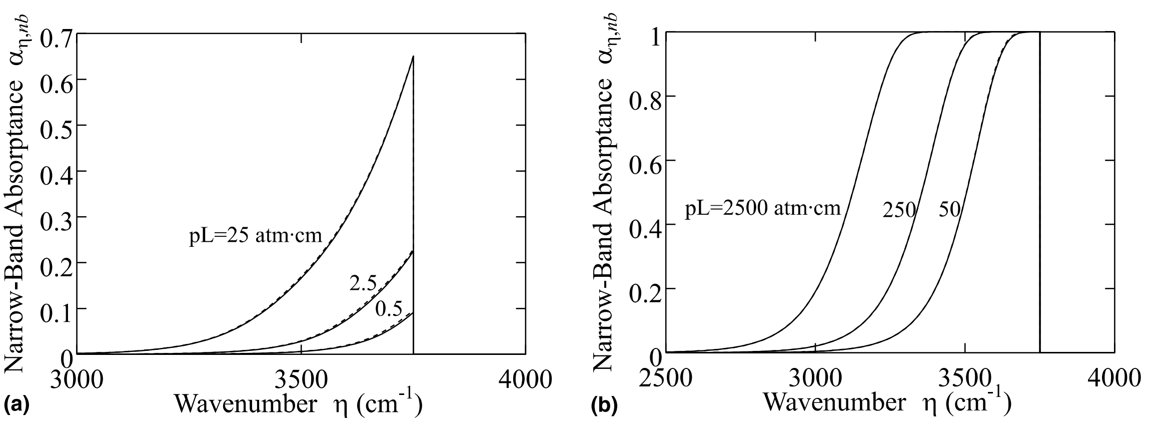


Fig. 9. Comparison of the narrow-band absorptance in a nonisothermal field at high or low pressure conditions (solid lines: base on the approach proposed in this paper, broken lines: based on the Curtis–Godson approximation): (a) CO_2:2.7 μm band ($p_{CO_2} = p_T = 0.05$ atm), (b) CO_2:2.7 μm band ($p_{CO_2} = p_T = 5$ atm).

treating narrow band nonuniformity in the nonisothermal field is satisfactory.

4. Treatment of narrow band nonuniformity in CO_2/H_2O mixture

In this section, we consider the gas mixture containing both CO_2 and H_2O, and pay attention to a spectral range where an absorption band of CO_2 overlaps with that of H_2O. In such spectral range, the energy emitted by CO_2 is absorbed by H_2O as well as by CO_2. Considering the radiation energy emitted by CO_2 from the No. i group of a specified narrow band, the absorption coefficient of CO_2 as an absorber can be determined with the concept explained above (Fig. 7). However, when this energy is absorbed by H_2O, all the groups in the relevant narrow band participate in the absorption process, because the position and interval among absorption lines of CO_2 do not agree with those of H_2O.

Hence, we assume that the sum of the energy allocated to each group in a narrow band of H_2O is equalized when the radiation energy emitted by CO_2 is transferred to the next layer at different temperature. In other words, dividing the energy belonging to the No. i group into N parts of equal amount, each of them is related to each of the absorption coefficients of H_2O evaluated for all the groups in the relevant narrow band. Therefore, the transmittance after the path length, L through the mixture containing CO_2 and H_2O is given by

$$\tau_{i,\eta,T}^{CO_2} = \frac{1}{N}\sum_{j=1}^{N} \exp\left\{-\left(\kappa_{i,\eta,T}^{CO_2}\cdot p_{CO_2} + \kappa_{j,\eta,T}^{H_2O}\cdot p_{H_2O}\right)L\right\}. \tag{18}$$

In addition, assuming that the path length of a beam of radiation energy across a single cell in the calculation mesh is not long, Eq. (18) is transformed as follows.

$$\tau_{i,\eta,T}^{CO_2} = \exp\left[-\left\{\kappa_{i,\eta,T}^{CO_2} p_{CO_2} + p_{H_2O}\cdot\frac{1}{N}\sum_{j=1}^{N}\kappa_{j,\eta,T}^{H_2O}\right\}L\right]. \tag{19}$$

Because the arithmetical mean of pressure absorption coefficient can be replaced by the effective ratio of line intensity to line interval $(S/d)_{\eta,T}$ which is given by the wide band model [11], the absorption coefficient used for tracing the absorption process of radiation energy emitted from the No. i group in a narrow band of CO_2 is given by

$$K_{i,\eta,T}^{CO_2} = \kappa_{i,\eta,T}^{CO_2} p_{CO_2} + (S/d)_{\eta,T}^{H_2O} p_{H_2O}. \tag{20}$$

Similarly, the absorption coefficient used for tracing the absorption process of radiation energy emitted from the No. i group in a narrow band of H_2O is given by

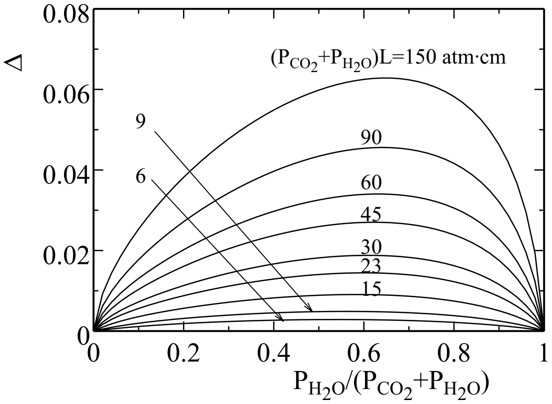


Fig. 10. Correction of total emittance due to coexistence of CO_2 and H_2O ($T = 1200$ K, $p_T = 1$ atm).

$$K_{i,\eta,T}^{H_2O} = \kappa_{i,\eta,T}^{H_2O} p_{H_2O} + (S/d)_{\eta,T}^{CO_2} p_{CO_2}. \tag{21}$$

Correction for the total emittance of CO_2/H_2O mixture due to the spectral overlap is usually expressed as follows

$$\varepsilon_{CO_2+H_2O} = \varepsilon_{CO_2} + \varepsilon_{H_2O} - \Delta\varepsilon. \tag{22}$$

Fig. 10 shows this correction $\Delta\varepsilon$ determined from the calculation based on the above-mentioned algorithm. Comparing this result with that in a reference book [16], it is recognized that the influence of spectral overlap can be incorporated properly by adopting the approach proposed in this paper.

5. Summary

(1) The error in a radiative transfer analysis induced by neglecting narrow band nonuniformity becomes more serious with the reduction in total pressure of gas mixture. Paying attention to the typical infrared-active species in combustion fields, such as water vapor and carbon dioxide, narrow band nonuniformity cannot be overlooked even on a typical condition of the atmospheric pressure.

(2) Transmittance of radiative energy averaged over a narrow band severely depends on the spectral pattern of incident power within a relevant narrow band when the narrow band nonuniformity is not negligible. It is illogical to carry out nongray analysis based on an algebraic formula which gives the transmittance averaged over a narrow band, because we can not formulate the narrow-band transmittance and power spectrum of transmitted energy for all the possible spectral pattern of incident energy. We cannot avoid further spectral division of each narrow band to incorporate the narrow band nonuniformity into radiative heat transfer analysis.

(3) A rational, explicit and practicable approach was proposed which can incorporate the narrow band nonuniformity into a nongray analysis of radiative heat transfer in nonisothermal infrared-active gas. This approach can be easily combined with many kinds of simulation technique developed for a gray analysis of radiative heat transfer, and, the computational load based on this approach is far lighter than that based on the line-by-line analysis. Rationality and plainness of this approach is secured by developing the approach based on a fact that the line center of an absorption line does not move with a change in temperature.

(4) By comparing the narrow-band transmittance of radiative energy passing through a stratified nonisothermal mass of infrared-active gas, it was confirmed that the transmittance evaluated based on the proposed approach is almost same as that evaluated based on Curtis–Godson approximation, and the proposed approach holds well even in nonisothermal field.

(5) An expansion of the proposed approach is tried to give this approach the applicability to the gas mixture containing more than one infrared-active species. Validity of expanded approach is confirmed in relation to the gas mixture containing water vapor and carbon dioxide.

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